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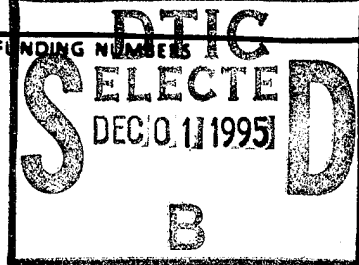
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It was proposed to seek experimental confirmation of theoretical predictions that the opening and closing of cracks in structures during vibration can provide dramatic qualitative and quantitative signatures of crack presence, even if the crack is very small. Cracked and uncracked laboratory-sized beams were driven by superpositions of two harmonic loads. Responses at the difference or sum of the two driving frequencies - theoretically indicative of nonlinearity, and large if that frequency is a natural frequency of the beam - were compared to the predictions of previously developed theory. The proposed technique, in which the beam is driven by the super-position of two harmonic signals whose difference or sum is equal to the natural frequency of the beam and the response is monitored at the natural frequency, was found, as expected, to be a good way of measuring nonlinearity. After laboratory background nonlinearities were reduced, it was found that the remaining nonlinearity of a cracked beam was substantially greater than that of the uncracked beams and that the proposed technique may be capable of detecting it and identifying its opening load. The work has implications for remote nondestructive evaluation of cracks in vibrating structures and in general for the structural dynamics of systems with contact nonlinearities. Further work is recommended in automating the tests and further exploring and minimizing the causes of baseline nonlinearity, and attempting to quantitatively characterize the nonlinear crack signatures.

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# **Nonlinear Vibrations of Cracked Beams**

**Final Report**

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**10 March, 1995**

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**Nonlinear Vibrations of Cracked Beams**

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**Abstract:**

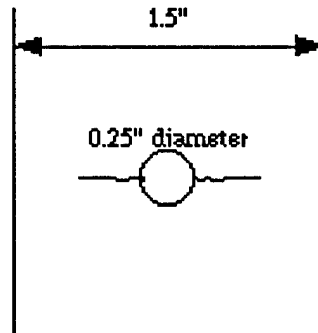
It was proposed to seek experimental confirmation of theoretical predictions that the opening and closing of cracks in structures during vibration can provide dramatic qualitative and quantitative signatures of crack presence, even if the crack is very small. Cracked and uncracked laboratory-sized beams were driven by superpositions of two harmonic loads. Responses at the difference or sum of the two driving frequencies - theoretically indicative of nonlinearity, and large if that frequency is a natural frequency of the beam - were compared to the predictions of previously developed theory. The proposed technique, in which the beam is driven by the super-position of two harmonic signals whose difference or sum is equal to the natural frequency of the beam and the response is monitored at the natural frequency, was found, as expected, to be a good way of measuring nonlinearity. After laboratory background nonlinearities were reduced, it was found that the remaining nonlinearity of a cracked beam was substantially greater than that of the uncracked beams and that the proposed technique may be capable of detecting it and identifying its opening load. The work has implications for remote nondestructive evaluation of cracks in vibrating structures and in general for the structural dynamics of systems with contact nonlinearities. Further work is recommended in automating the tests and further exploring and minimizing the causes of baseline nonlinearity, and attempting to quantitatively characterize the nonlinear crack signatures.

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## Nonlinear Vibrations of Cracked Beams

### Activity Report:

Steel and aluminum beams of dimensions 1.5" x .25" x 30" were subjected to fatigue loads in order to grow cracks for vibration testing. The mild steel chosen was found to be too ductile, however, and no significant cracks were created by the fatigue loads before permanent plastic deformations ruined the specimen. In an aluminum alloy, 7075-T6, however, it was found that fatigue cracks could be grown from a stress concentrating 0.25" circular hole in a few days (at 5 load cycles/second) of four-point bending fatigue. Loads were of the order of 30% of the nominal yield strength. At late times, the crack could be seen by eye during the opening part of the load cycle. The depths of the cracks so generated were not quantified, but they are estimated to be far less than a quarter of the beam thickness, since the cracks (shown in Figure 1) radiating out of the drilled hole were only about half-way to the edge of the beam. The crack depth could presumably be measured *ex post facto*, after the vibration testing, but destroying the specimens at this point would probably be premature.



**Figure 1. Top view of test specimen with fatigue cracks shown radiating away from the stress concentration hole**

The vibration testing phase has had many stages, as baseline nonlinearities, both mechanical and electronic, and the basic nature of a fatigue crack nonlinearity were explored.

In the first tests the beam was simply clamped to a large shaker head, with the fatigue crack located near the clamp, i.e., near the point of maximum bending stress. The shaker was driven, as described in the proposal, by a superposition of two independently generated harmonic signals. Cracked and uncracked specimens were observed to have very weak nonlinearity, and, furthermore, no significant difference in their nonlinearities. The absence of nonlinearity in the uncracked specimens was good news, because it meant that the power amplifier and shaker and driver circuit were found to be linear. The absence of nonlinearity in the cracked beam, it was hypothesized, was due to the crack not opening (or not closing, in any case not changing its state) during the vibratory motion. One could compensate for this by increasing the shaker displacement amplitude, but the necessary shaker loads were calculated to be well above the level at which harmonic distortion would occur, either in the shaker itself or in the power amplifier. This kind of electronic or driver nonlinearity would probably mask the mechanical nonlinearity.

It was therefore concluded that the tester needed to be able to vary an applied static load that could bring the crack to the verge of opening. The far end of the beam, far from the shaker head, was therefore clamped to a vise that ran on a vertical track whose height could be adjusted with a hand crank. This substantially stiffened the system and increased its dissipation, lowering the Q factor. The shaker was thus required to apply loads that would induce driver nonlinearities. New cracked and uncracked beams, of greater length (about 70"), were then fabricated to lower the system stiffness.

The new systems were found to have very large mechanical nonlinearities, especially at the larger static loads. These nonlinearities were large enough to mask any nonlinearity due to the crack. They were found to be due to the large dynamic tensile stresses induced in the beams as they underwent simple vibrational bending. As the shaker end of the beam vibrates vertically, the far end, clamped as it is at the vise, cannot retract longitudinally, thus the length of the beam is slightly increased - leading to large tensile loads, which in turn increase the natural frequency of the bending motions (much as compressive loads can lower the natural frequency of bending and, if strong enough, lower them to zero leading to buckling). The hypothesis was confirmed by the great increase in natural frequency of bending with applied static load.

For this reason the mounting needed to be redesigned once more. Two ways to mount the beam such that large static loads could be applied in such a way that the beam would be free to retract longitudinally were considered. Due to time limitations, only one of these two redesigns was pursued. One could have applied the static load by means of a long stiff spring attached at the end opposite the shaker. If the spring is long enough, it will rotate as the end retracts, but it will not add any significant extra load as the end retracts. This design was not implemented. Instead the vise and vertical track were left as in the previous design, but rollers were added to the clamp at the shaker head, as shown in Figure 2. A tendency for tensile loads in the beam then, in principle, allows the beam to retract at the shaker head by sliding on the rollers. No tensile loads should develop.

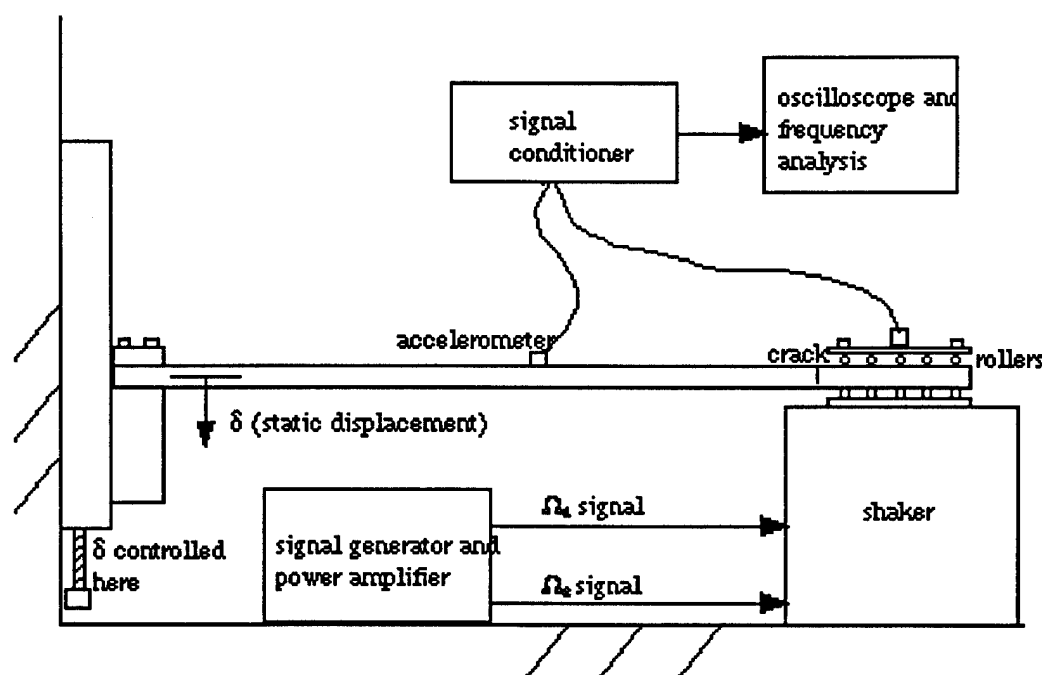


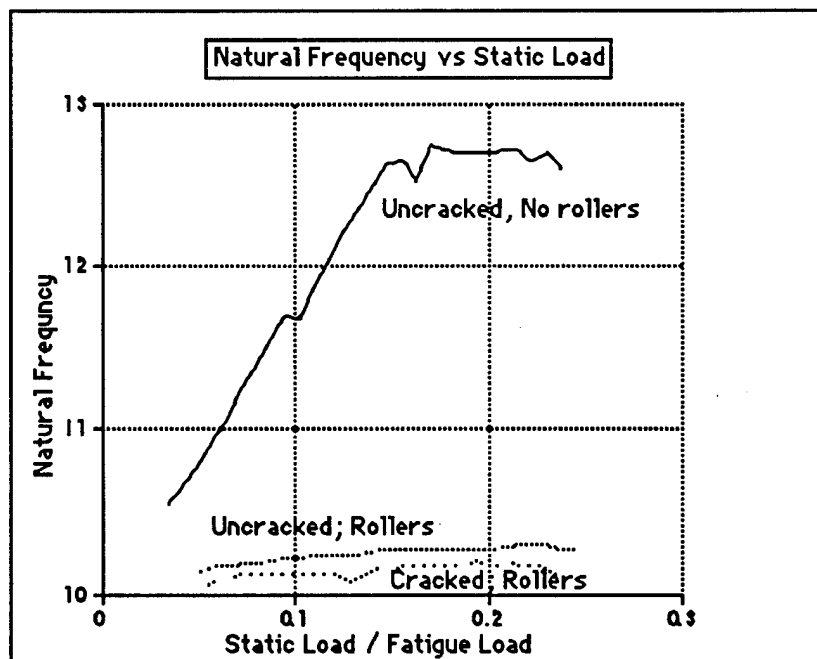
Figure 2. Side view of experimental set-up

The testing procedure began with starting at some particular static load level and finding the natural frequency of the beam by slowly sweeping the frequency of a harmonic excitation and monitoring the ratio of the amplitude of the shaker head acceleration to that of the midpoint of the beam. The transfer function at the resonant frequency could then also be recorded and a nominal "Q" ascertained. The beam was then forced at two frequencies such that the sum was the natural frequency and such that one frequency remained fixed. After waiting for the transient to decay, a Fourier analysis upon the steady-state time history yielded the energies associated with the three frequencies of interest. This data was recorded, and then the cycle was

repeated after incrementing the static load by a small amount. The procedure was slow and tedious. The right equipment could allow us to automate the procedure, thereby speeding the process and reducing the possibility of operator error. In the short project duration reported here, however, we judged it more efficient to generate data that could be scrutinized in detail than to spend our time and effort designing an automated process.

The design that utilized the rollers at the shaker head was more successful. Baseline nonlinearities in the uncracked beams, while not eliminated, were much smaller. The changes in natural frequency with static load were greatly reduced by the new design, as shown in Figure 3. Figure 4 shows nonlinear signature versus static load for both a cracked and an uncracked beam. As anticipated the nonlinear signature was strongest at a particular degree of static load - which could then perhaps be identified with the crack opening load. Three additional runs were conducted in which the nonlinear signature was measured as a function of static load for the same cracked beam. Figure 5 shows a less distinct difference between the nonlinear behavior of the two beams. The implication is that the method is, so far, not as repeatable as one would prefer. It is expected that there are still baseline mechanical nonlinearities associated with the clamping and, especially, with the rollers.

Figure 6 shows the results of sweeping over a larger range of static bending moment while testing a cracked beam. Negative static loads corresponded to crack closure, while positive loads tend to open the crack. In this particular run, we see that the crack-opening side appears to be in some general sense more nonlinear than the crack-closing side, but the baseline nonlinearities are strong enough here to obscure the issue. No single static load seems a likely candidate to be identified as the opening load.



**Figure 3. The use of rollers at the shaker head clearly reduces the baseline nonlinearity**

Figure 7 shows the most promising results. Here a very prominent nonlinear spike can be associated with a particular static load. The presumed opening load is, however, different than the one that would be obtained by examining Figure 4. The effect may be spurious. In any case, as of the date of this report, this scan has not been completed.

Most nonlinearities are of quadratic or higher exponent in the vibration amplitudes. The low exponent theoretically associated with a crack implies a unique character to the amplitude dependence of the associated nonlinear effect. This amplitude dependence has not yet been investigated experimentally.

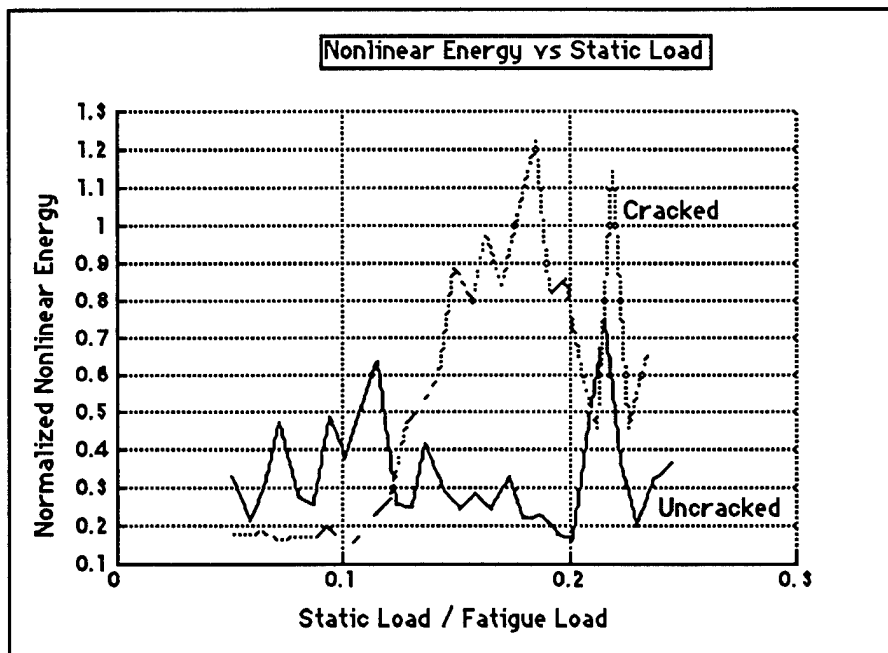


Figure 4. Here the cracked beam clearly exhibits a greater nonlinear signature than does the uncracked beam. The point of maximum nonlinearity is tentatively associated with the opening load.

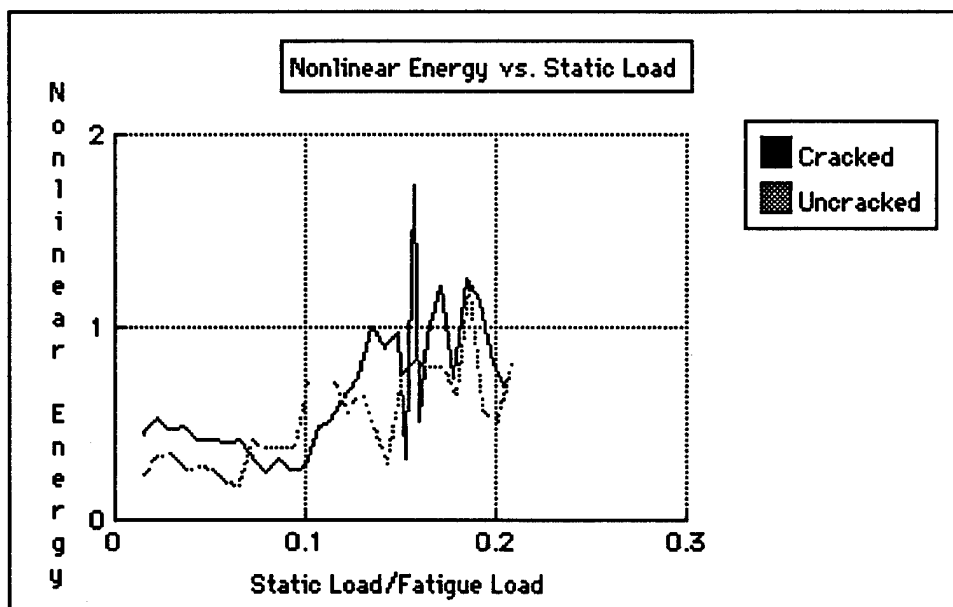


Figure 5. In this run the nonlinear behavior of the uncracked beam is somewhat greater than it was in the run reported in figure 4. The cracked beam retains though, its character of greater nonlinearity.

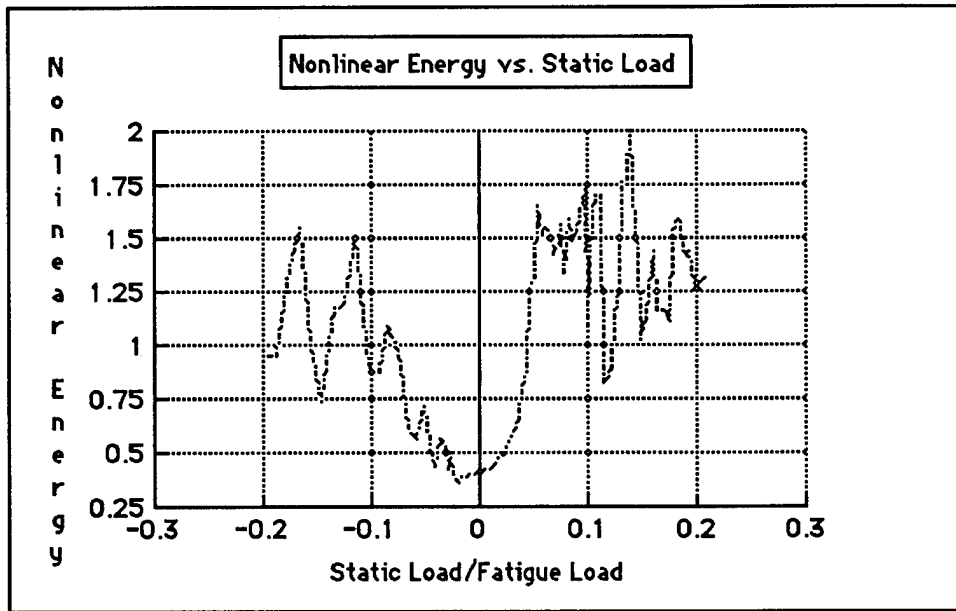


Figure 6. Here the nonlinear behavior of the cracked beam is examined over a wider range of static load.

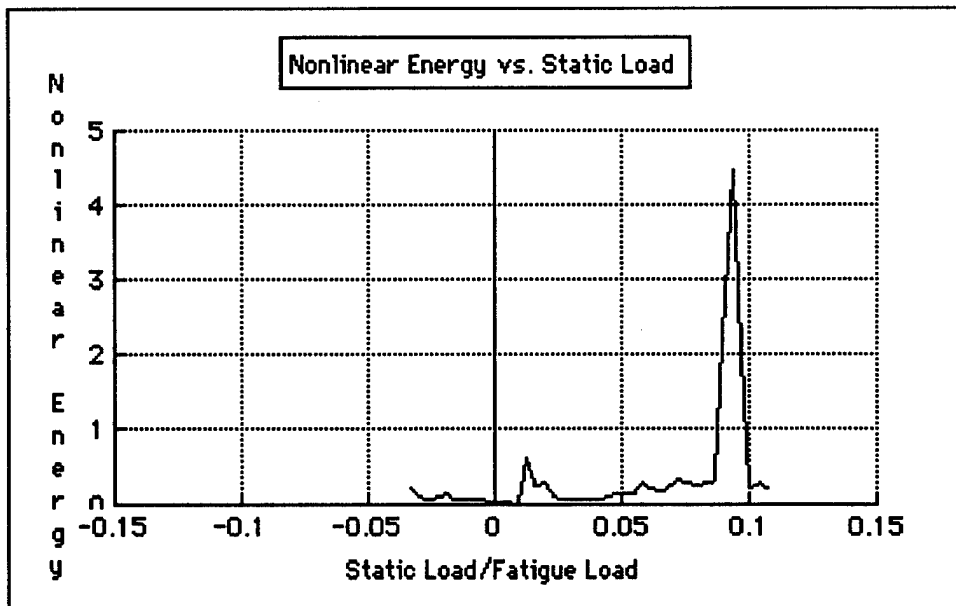


Figure 7. This cracked beam shows a very prominent nonlinear spike which may be associated with a sharply defined opening load.



Much of this work has been presented at the recent Acoustical Society of America meeting in Austin, Texas. [J. Sundermeyer and R. Weaver, *J. Acoust. Soc. Am.* 96, 3322 (1994)]. An archival journal publication and a Ph.D. thesis are in preparation.

#### Conclusions:

The method has been shown to be extremely effective in detecting the presence of small mechanical nonlinearities. While most of the experimental results show an obvious difference between the nonlinear behavior of cracked and uncracked beams; the baseline nonlinearities, which are still fairly strong and erratic, may be masking the crack signature. Further refinement of the experimental design, such as the use of the long stiff springs mentioned earlier or perhaps modification of the roller bearings at the shaker head, will be necessary in order to isolate the nonlinear signature due solely to the opening and closing of the fatigue crack. Increased automation of the testing procedures would reduce tedium and the associated tendency for operator error. The resulting increase in speed would also allow for the exploration of other testing parameters.

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No inventions have resulted from the work done in this project.

One publication, the abstract of a meeting presentation, has, so far, resulted: J. Sundermeyer and R. Weaver, *J. Acoust. Soc. Am.* 96, 3322 (1994). Others are in preparation and will be submitted as they become available.

J Sundermeyer expects to receive his PhD this summer based, in part, upon work done in this project. A copy of this thesis will be submitted when it becomes available.

R Weaver and J Sundermeyer are the only personnel who have been supported by this contract.